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Determining the Average R-Value of Tapered Insulation

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ABSTRACT

Heat loss through tapered roof insulation is generally computed using an R-value based on the average thickness of the tapered section. However, this misrepresents the actual performance of the tapered insulation, which is always less efficient than an equal volume of untapered insulation. For commonly encountered slopes formed with one-way and four-way tapers, the true efficiency of tapered insulation (compared with an equal volume of untapered insulation) depends only on the ratio of high- and low-point R-values, and ranges from 100% to about 70% for R-value ratios between 1 and 10. The impact of curved heat flow trajectories on the efficiency of the tapered forms is a function of taper angle or slope, and becomes significant only at slopes much steeper than those found in typical tapered roof insulation applications. Equations are derived, and tables are presented, for the efficiency of tapered insulation considering volumetric forms typically encountered in practice. Examples illustrate how these tools can be used to accurately calculate heat loss through a roof assembly with tapered insulation.

INTRODUCTION

It is commonly assumed that the average R-value of tapered insulation is equivalent to the R-value of its average thickness (Graham 1995; PIMA). However, because heat loss through insulation is *inversely* proportional to insulation thickness, a unit increase or decrease of insulation thickness does not result in a constant increase or decrease in heat loss: a unit change in thickness (assuming uniform R-value) from 4 to 3 results in an increase in heat loss of 1/3 divided by 1/4, or 133%; whereas a unit change in thickness from 3 to 2 results in an increase of 1/2 divided by 1/3, or 150%. As the material gets thinner, “energy consumption,” or heat flux, gets bigger at an increasing rate (Johnson 2009).

For this reason, heat loss through tapered insulation is not equivalent to heat loss through the same quantity (volume, or average thickness) of constant-thickness insulation. Insulation having a simple taper will be less efficient than insulation with the same volume configured with constant thickness, since the portion of tapered insulation that is *thinner* than average will lose more heat than the *thicker* part will save.

Several tapered insulation forms are commonly used, including one-way slopes, two-way slopes, and four-way or pyramidal (along with inverse pyramidal) shapes. So-called crickets are often placed above a one-way or two-way slope to direct water to drains. Cricket geometry can be resolved into one or more triangular solids having different thickness at each vertex. While crickets may be manufactured as separate pieces of insulation placed on top of tapered insulation with one- or two-way slopes, they will be analyzed as if they extended vertically to an assumed horizontal plane (roof deck). These tapered roof geometries are illustrated in Figure 1.

Certain geometries have the same underlying efficiency and need not be separately analyzed: two-way slopes may be analyzed as two one-way slopes; one-way slopes that converge upward to a point have the same efficiency as pyramids (four-way slopes with external drainage); and one-way slopes that converge downward to a point have the same efficiency as inverted pyramids (four-way slopes with internal drainage).

For the same reason that the efficiency of tapered panels is not the same as that of flat panels with equal volume, one cannot simply add the equivalent average R-value, computed for an isolated piece of tapered insulation, to R-values computed for other elements of the roof assembly (e.g., roof

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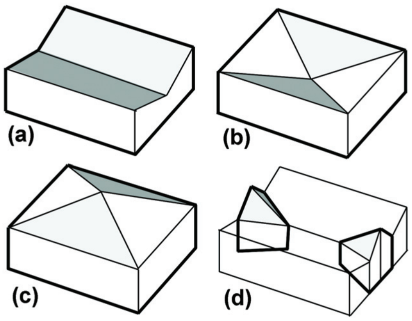

Figure 1 *Common tapered insulation forms include (a) one-way and two-way slopes, (b) four-way slope with internal drainage, (c) four-way slope with external drainage, and (d) crickets.*

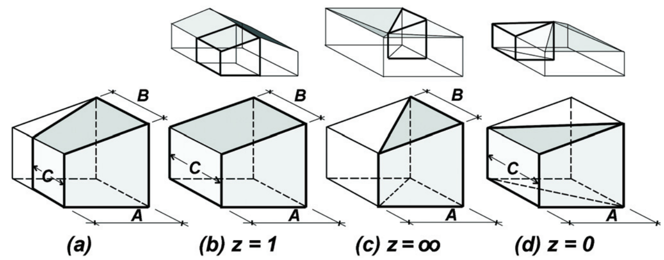

Figure 2 *All commonly encountered taper geometries can be derived from the consideration of the trapezoidal solid (a). For the ratio of sides* B/C = z *= 1 (b), we get a one-way or two-way slope; for* z *= ∞ (c), we get a one-way slope, converging downward to a point or a four-way slope with internal drainage; and for* z *= 0 (d), we get a one-way slope converging upward to a point or a four-way slope with external drainage.*

deck, interior finishes, other insulation, air films). Adding material with constant R-value to material with variable R-value (e.g., to a piece of tapered insulation) changes the ratio of overall thickness (or R-value) upon which the efficiency of the tapered panel is based. Therefore, it is necessary to consider the entire roof assembly when analyzing any piece of tapered insulation. Where the terms thickness or R-value are used in the following discussion, they always refer to the *total* R-value of the entire roof assembly, and not just the portion consisting of the actual tapered insulation. In general, insulation thickness may be substituted for R-value when only a single material with uniform R-value is used; otherwise, the total R-value must be explicitly computed.

There are two main strategies for determining the efficiency of tapered insulation. One strategy is to compare the heat flux of the tapered panel to that of an untapered (flat) panel having the same volume. This method gives a true measure of the efficiency of the tapered panel, ε_t, but may be cumbersome to use in practice, since it requires the calculation of a *true* average thickness for the tapered panels. A more practical strategy is to find the efficiency, ε_a, based on an *assumed* "average" panel thickness. Letting H equal the thickness (or R-value) at the high point of tapered insulation and L equal the thickness (or R-value) at the low point of tapered insulation, this assumed average thickness (or R-value) is $(H + L)/2$.

For a one- or two-way taper, there is no difference between these two strategies since the *assumed* average and *true* average thicknesses are the same; however, using an assumed average thickness of $(H + L)/2$ is much easier for all other taper geometries, since information about high- and low-point thicknesses is readily available. Derivation of the efficiencies ε_a and ε_t for various taper geometries follows.

EFFICIENCY OF TAPERED INSULATION

The following tapered forms will be considered, based on the geometries illustrated in Figure 1: one-way taper, four-way taper (sloping both to interior drain and to exterior drain); and triangular cricket. In these derivations, let:

A = length of tapered panel measured horizontally, in the direction of taper slope

B = width of tapered panel at high point

C = width of tapered panel at low point

E = heat flux at thickness, L

H = thickness (or R-value) at high point of tapered insulation or cricket

L = thickness (or R-value) at low point of tapered insulation or cricket

M = thickness (or R-value) at intermediate height of cricket

e = heat flux at any point

f = ratio of high- to low-point thickness (R-value) of taper, H/L

g = ratio of intermediate- to low-point thickness (R-value) of cricket, M/L

z = ratio of panel width at top and bottom of taper, B/C

Except for the cricket, which is analyzed separately, the efficiency of all these geometric forms can be derived from consideration of a single trapezoidal solid, shown in Figure 2a. By altering the ratio of high- and low-point sides, $z = B/C$, we can find the efficiencies of one- and two-way slopes (Figure 2b, where $z = 1$); one-way slopes converging downward to a point or four-way slopes with internal drainage (Figure 2c, where $z = \infty$; and one-way slopes converging upward to a point or four-way slopes with external drainage (Figure 2d, where $z = 0$).

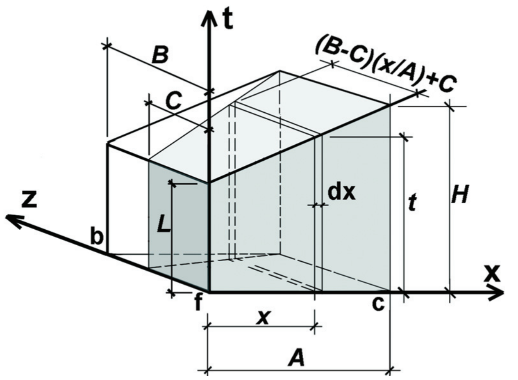

Figure 3 *Geometry of one-way tapered insulation with trapezoidal base.*

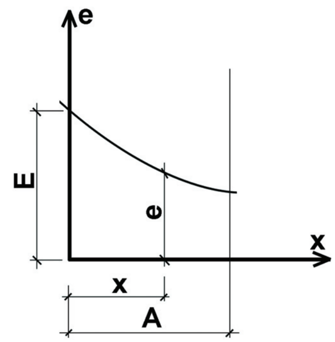

Figure 4 *Heat flux, e, at any point along tapered insulation, where* E *is the heat flux at thickness* L.

One-Way Trapezoidal Taper with Parallel High- and Low-Point Sides

1. From Figure 3, the thickness, t, of the trapezoidal solid at any distance, x, is the height of the line connecting (0, L) and (A, H):

$$t = \left(\frac{H-L}{A}\right)x + L \tag{1}$$

2. The heat flux, e, at any point, inversely proportional to the R-value or thickness of the material at that point (see Figure 4), is

$$e = E\left(\frac{L}{t}\right) , \tag{2}$$

where E is the heat flux at thickness equal to L.

3. Substituting t from Equation 1 into Equation 2, we get

$$e = \frac{EL}{\frac{(H-L)}{A}x + L} = \frac{E}{\frac{(H-L)}{LA}x + 1} . \tag{3}$$

4. Multiplying by the insulation panel width, $(B-C)(x/A) + C$, and integrating Equation 3 over length A to find the total heat flux V of the panel, we get

$$V = \int_0^A \frac{EC}{A}\left(\frac{(z-1)x + A}{\frac{(f-1)}{A}x + 1}\right)dx , \tag{4}$$

where $f = H/L$ and $z = B/C$.

5. The solution to Equation 4 is

$$V = EAC\left\lfloor \frac{(f-z)\ln(f) + (z-1)(f-1)}{(f-1)^2} \right\rfloor . \tag{5}$$

6. The total heat flux, W, for a trapezoidal untapered panel with constant thickness of $(H + L)/2$, sides B and C, and length, A, is

$$w = \frac{EAC(z+1)}{(f+1)} . \tag{6}$$

7. The *assumed* efficiency, ε_a, of the trapezoidal tapered insulation—compared with an un-tapered block of thickness $(H + L)/2$—is found by dividing Equation 6 by Equation 5, and multiplying by 100 to express the following efficiency as a percentage:

$$\varepsilon_a = \frac{100(z+1)(f-1)^2}{(f+1)[(f-z)\ln(f) + (z-1)(f-1)]} \tag{7}$$

8. The actual average thickness, t, for the one-way trapezoidal panel, computed by dividing the volume by the area rather than using $(H + L)/2$, is

$$t = (2L)\frac{(f-1)(z-1)/3 + (z+f)/2}{(z+1)} \tag{8}$$

and the *true* efficiency, ε_t, becomes

$$\varepsilon_t = \frac{25(z+1)^2(f-1)^2}{[(f-1)(z-1)/3 + (z+f)/2][(f-z)\ln(f) + (z-1)(f+1)]} \tag{9}$$

The efficiencies of one-way slopes, four-way slopes with internal drains, and four-way slopes with external drains are all variations of Equations 7 and 9, found by setting $z = 1$, $z = \infty$, and $z = 0$, respectively. The resulting efficiency expressions are summarized in Table 1. Typical efficiency values for these taper geometries are found in Table 2.

It can be seen that tapered insulation efficiency depends only on the variable f, which, in turn, depends only on the ratio of the thicknesses (or R-values) at the high- and low-points of the tapered panel. However, there are two caveats. First of all, the panel cross section cannot be triangular (with the thickness at one end equal to zero)—as this would lead to the impossible

Table 1. Efficiency of Tapered Insulation[1]

	ε_a: Compared with Flat Panel Having Assumed Average Thickness (R-Value) of $(H + L)/2$	ε_t: Compared with Flat Panel Having True Average Thickness (R-Value) of Volume/Area
One-way slope (or two-way slope)	$\varepsilon_a = \dfrac{200(f-1)}{(f+1)\ln(f)}$	$\varepsilon_t = \dfrac{200(f-1)}{(f+1)\ln(f)}$
Four-way slope with interior drain (or one-way slope converging down to a point)	$\varepsilon_a = \dfrac{100(f-1)^2}{(f+1)[f-1-\ln(f)]}$	$\varepsilon_t = \dfrac{150(f-1)^2}{(2f+1)[f-1-\ln(f)]}$
Four-way slope with exterior drain (or one-way slope converging up to a point)	$\varepsilon_a = \dfrac{100(f-1)^2}{(f+1)[f\ln(f)-f+1]}$	$\varepsilon_t = \dfrac{150(f-1)^2}{(f+2)[f\ln(f)-f+1]}$
Triangular cricket[2]	$\varepsilon_a = \dfrac{\varepsilon_1\varepsilon_2}{\varepsilon_2\left(\frac{f-g}{f-1}\right)+\varepsilon_1\left(1-\frac{f-g}{f-1}\right)}$	$\varepsilon_t = \varepsilon_a \times \dfrac{1.5(f-1)(f+1)}{(f+2g)(f-g)+(1+2g)(g-1)}$

1. Efficiencies based on $f = H/L$ and (for crickets only) $g = M/L$, where H, L, and M are thicknesses (R-values) at high, low, and intermediate points, respectively.
2. For cricket efficiency parameters, ε_1 and ε_2, see Equations 10 and 11.

condition of “infinite” heat flux. Secondly, the slope of the tapered panel cannot be too great; in that case, the basic assumption underlying this method—that the heat flux at each point is inversely proportional to its vertical thickness at that point—becomes problematic, as the pattern of heat loss through the sloping surface would be more complex than what was assumed. This concern is addressed in more detail below.

Neither caveat affects typical tapered insulation panels. In the first case, the total R-value through any point on the insulation cannot be zero, since the R-value of the insulation does not exist by itself, but must be measured together with the R-value of all other adjacent material layers, including air films and any substrate or interior finishes. In the second case, the slopes of tapered insulation are typically in the range of 1/8 in. per foot (1:96) up to 1 inch per foot (1:12), values that are essentially flat for the purposes of this discussion.

Cricket

Crickets are peculiar instances of tapered insulation typically placed on one- or two-way slopes to direct water to drains (see Figures 1*d* and 5).

While the actual cricket may literally rest on the sloping surface(s) formed by other pieces of tapered insulation, narrowing to zero thickness at its low point, our cricket is shown as if it extended downward to the horizontal roof deck, so that a triangular cricket element has different, non-zero thicknesses at each vertex. For example, triangle *a-b-c* (Figure 5) has a low-point thickness, L, and an intermediate thickness, M, at the perimeter; as well as a high-point, H, at the center. Crickets may be configured with one, two, or four of these triangular pieces, but, owing to their symmetry, it is sufficient to examine a single triangle.

Each triangle is analyzed by dividing it into two parts bounded by the line *b-f* (Figure 5) so that each of the triangular solids thus obtained has a rectangular (and not a trapezoidal) side, as shown in Figure 6. The efficiencies computed for each part, based on an assumed average thickness of $(H + L)/2$, are as follows:

$$\varepsilon_1 = \frac{100g\left(\frac{f}{g}-1\right)^2}{(f+1)\left[\frac{f}{g}\ln\left(\frac{f}{g}\right)-\left(\frac{f}{g}\right)+1\right]} \tag{10}$$

$$\varepsilon_2 = \frac{100(g-1)^2}{(f+1)[g-1-\ln(g)]} \tag{11}$$

where $f = H/L$ and $g = M/L$.

The efficiency of the whole cricket, derived from these partial values, is shown in Table 1. Values for cricket efficiency at different thickness (R-value) ratios are shown in Table 3.

What, then, is the loss of efficiency for tapered insulation assemblies in typical conditions? Considering tapered insulation with a one-way slope of 1/4 in. per foot (1:48) over a panel length of 40 ft (12.2 m), the *change* in thickness is 10 in. (250 mm). Where the slope and panel length are fixed as in this example, efficiency is a function only of the high- or low-point thickness (either ignoring or including any additional constant R-values within the roof assembly). From Table 2, for a low-point thickness of 2 in. (50 mm) and, therefore, a high-point thickness of 12 in. (300 mm), the ratio of thicknesses is $f = 6$, and the assumed efficiency, ε_a, is 80%. If the low-point thickness is doubled to 4 in. (100 mm), with the high-point thickness equal to 14 in. (350 mm), the ratio of thicknesses is $f = 3.5$, and ε_a increases to 89%. For a low-point thickness of 8 in.

Table 2. Typical Efficiency Values (Percent) of Tapered Insulation Compared with Panels of Constant Thickness (R-Value)[1]

	Ratio of R-Values at High and Low Points (max/min)									
	1	2	3	4	5	6	7	8	9	10
One-way slope	100 (100)	96 (96)	91 (91)	87 (87)	83 (83)	80 (80)	77 (77)	75 (75)	73 (73)	71 (71)
Four-way slope with interior drain (or one-way slope converging down to a point)	100 (100)	109 (98)	111 (95)	112 (93)	112 (91)	111 (90)	111 (89)	111 (88)	110 (87)	110 (86)
Four-way slope with exterior drain (or one-way slope converging up to a point)	100 (100)	86 (97)	77 (93)	71 (88)	66 (85)	62 (82)	59 (79)	57 (76)	54 (74)	53 (72)

1. Efficiencies, ε_a, are based on assumed average thicknesses (R-values) equal to high- plus low-point values divided by two. Values shown in parentheses, ε_t, are based on true average values.

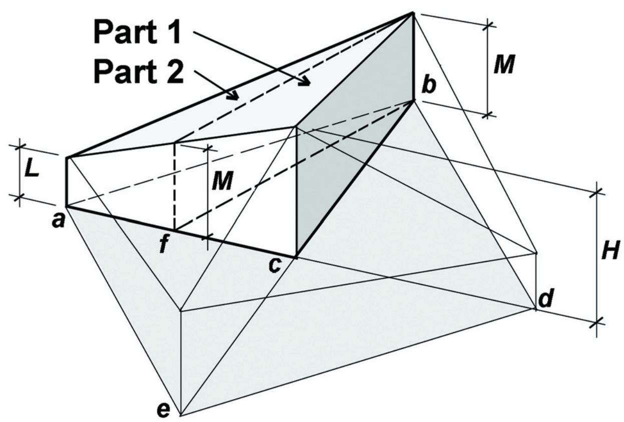

Figure 5 *Geometry of cricket.*

(200 mm) and a high-point thickness of 18 in. (450 mm), the ratio of thicknesses is $f = 2.25$, and ε_a increases to 95%. In general, ε_a for tapered insulation with a one-way slope ranges from 100% to 71% as the ratios of thickness or R-value go from 1 to 10 (see Table 2).

For four-way slopes with exterior drains, the range of assumed efficiencies is even more extreme—reaching a value of 52% at a ratio of 10:1. For four-way slopes with interior drains, the opposite tendency occurs, with assumed efficiencies as high as 112%. This does not mean that such geometries are actually more efficient than using flat panels with the same volume of material; the *true* efficiencies for all taper geometries (shown in parentheses in Table 2) are always less than 100%. However, it does suggest that using an average thickness as a basis for computing the R-value of tapered insulation—whether that thickness is taken as the arithmetic mean of high and low points, or as a true average calculated by dividing the total volume of insulation by its area—can seriously misrepresent the actual heat loss through the roof assembly.

TEMPERATURE GRADIENT WITHIN TAPERED INSULATION

Up until now, our model of heat flow through the tapered insulation has assumed that heat flows vertically through the insulation, so that heat flux can be taken as being inversely proportional to the thickness of the insulation at any point, measured as a vertical height. In reality, heat flows at right angles to the temperature gradient, which remains constant for steady-state conditions (i.e., for constant interior and exterior temperatures on either side of the tapered insulation panel). As shown in Figure 7, the temperature gradient lines for tapered insulation converge at the center of a circle with radius r. Since heat flow trajectories are at right angles to these gradient lines, the trajectories are circular arcs of length c. The curved length c is found by equating the ratio c/θ with the ratio of the circle's circumference to its measure in radians, $2\pi r/2\pi = r$. We get $c/\theta = r$, or

$$c = r\theta \ , \tag{12}$$

where θ is the angle of the taper expressed in radians.

The vertical thickness, t, can also be expressed in terms of the angle, θ, where

$$t = r\tan\theta \ . \tag{13}$$

The ratio of assumed vertical heat flow trajectory length to actual curved trajectory length is therefore t/c, and the ratio of heat loss (heat flux) is c/t, since heat loss is inversely proportional to trajectory length. Dividing c from Equation 12 by t from Equation 13, we get a measure of the relative change in heat loss when the assumed vertical, rather than the actual curved, trajectory is considered, as follows:

$$\frac{c}{t} = \frac{r\theta}{r\tan\theta} = \frac{\theta}{\tan\theta} \tag{14}$$

where, as before, θ is the angle of the taper expressed in radians.

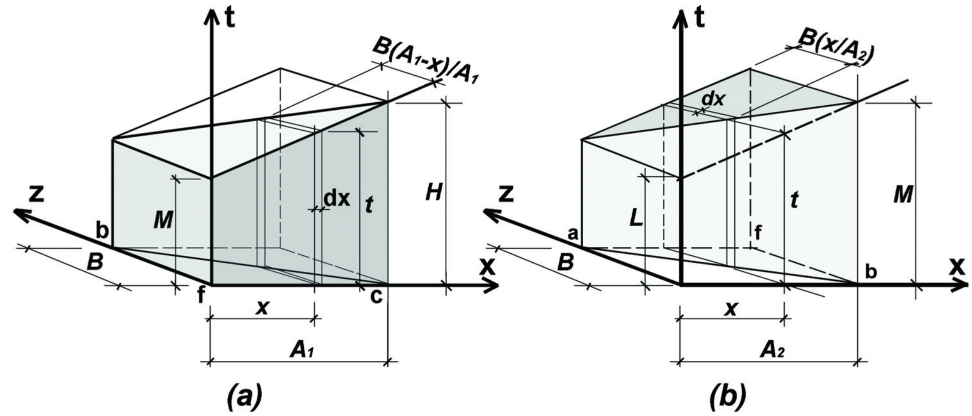

Figure 6 *Geometry of cricket: (a) upper part and (b) lower part.*

Table 3. Typical Efficiency Values (Percent) of Crickets Compared with Panels of Constant R-Value[1]

Ratio of R-Values at Intermediate and Low Points (int/min)	Ratio of R-Values at High and Low Points (max/min)									
	1	2	3	4	5	6	7	8	9	10
2	—	—	96 (96)	87 (93)	80 (90)	75 (87)	71 (85)	67 (83)	64 (81)	62 (79)
3	—	—	—	100 (94)	92 (92)	85 (90)	80 (88)	76 (86)	73 (84)	70 (82)
4	—	—	—	—	102 (92)	95 (90)	89 (89)	84 (87)	80 (86)	77 (85)
5	—	—	—	—	—	103 (90)	97 (89)	91 (88)	87 (87)	83 (86)
6	—	—	—	—	—	—	104 (89)	98 (88)	93 (87)	89 (86)
7	—	—	—	—	—	—	—	105 (88)	99 (88)	95 (87)
8	—	—	—	—	—	—	—	—	105 (87)	100 (87)
9	—	—	—	—	—	—	—	—	—	105 (87)

1. Efficiencies, ε_a, are based on assumed average R-values equal to high- plus low-point values divided by two. Values shown in parentheses, ε_t, are based on true average values.

As can be seen in Table 4, this difference in heat loss between the actual and assumed trajectories is inconsequential for taper angles typically encountered in practice: neither a slope of 1/4 in. per foot (1:48 or 1.2°), a slope of 1/2 inch per foot (1:24 or 2.4°), nor a slope of 1 inch per foot (1:12 or 4.8°) will have any appreciable effect on the calculation of heat loss. Only at taper angles greater than about 9° does the relative change in calculated heat loss become more than 1%, and only at angles greater than 30° does this change become more than 10%.

TAPERED INSULATION EXAMPLE 1

For a simple one-way or four-way slope, the true heat loss through a roof with tapered insulation can be found by multiplying the R-value calculated using an assumed average thickness, $(H+L)/2$, by the appropriate efficiency coefficient found in Table 2. For the simple four-way taper with exterior drains shown in Figure 8, neglecting any other roof materials that might add additional R-value, the ratio of high- and low-point thickness is 12/2 = 6. From Table 2, the efficiency of the tapered insulation, $\varepsilon_a = 62\%$. For this example, we will assume an R-value of 6°F·ft^2·h/Btu per inch of insulation material

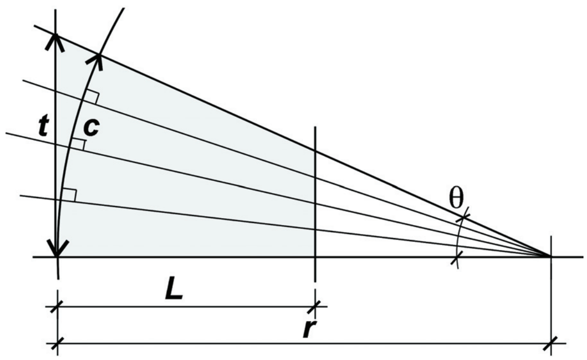

Figure 7 *Section through tapered insulation showing assumed vertical trajectory (length = t) and actual circular trajectory (length = c).*

(0.0416 K·m^2/W/mm) and a temperature differential of 70°F (36.8°C). The assumed average thickness of the insulation is $(H + L)/2 = (12 + 2)/2 = 7$ in. (178 mm), so the true average R-value is 6 × 7 × 0.62 = 26.04 (rather than 6 × 7 = 42, as would be the case if the *efficiency* of the tapered shape were not considered). The true average U-factor is therefore 1/26.04 = 0.0384 and the total heat loss over the entire roof area of 40 × 40 or 1600 ft2 (149 m2) is 0.0384 × 1600 × 70 = 4301 Btu/h (1260 W). Using the "average" height of 7 in. (178 mm) without accounting for the efficiency of the tapered insulation would result in an assumed heat loss of (1/42)(1600 × 70) = 2667 Btu/h (782 W), a value that is 62% lower than the true heat loss.

TAPERED INSULATION EXAMPLE 2

A more complex roof is shown in Figure 9. The primary one-way tapers have a slope of 1/4 in. per foot (1:48) with a high-point thickness of 6.5 in. (165 mm) and a low-point thickness of 0.5 in. (13 mm). The intermediate thickness at the cricket (1/3 of the way between the low- and high-point of the one-way taper) is 2.5 in. (64 mm). The high-point of the cricket is set at 4.5 in. (114 mm). It is assumed that the R-value of the tapered insulation is 6°F·ft^2·h/Btu per inch (0.0416 K·m^2/W/mm) and that other elements of the roof assembly (e.g., concrete deck, air films) contribute an additional R-value of 1°F·ft^2·h/Btu (0.176 K·m^2/W). The temperature differential is assumed to be 70°F (36.8°C).

For such complex geometries, it is useful to tabulate efficiencies for the individual components of the roof, as shown in Table 5.

In this example, the true heat loss through the roof assembly, found in row *q* of Table 5, is 12,429 Btu/h (3642 W). This value is approximately 13% greater than the value of 11,007 Btu/h (3226 W) that would be computed using an assumed average thickness for the tapered insulation—i.e., based on $(H + L)/2$—calculated in row *r* of Table 5. If the *true* average thickness were used—equal to the total volume of insulation divided by the roof area—the assumed heat loss would be lower, and therefore even less accurate than the true heat loss.

SUMMARY AND DISCUSSION

Tapered insulation is less efficient than the same volume of insulation configured with no slope. For low slopes typically encountered in practice—in the range of 1 in./ft (1:12), 1/2 in./ft (1:24), 1/4 in./ft (1:48), or 1/8 in./ft (1:96)—the loss of efficiency is a function only of the ratio of high- and low-point panel thickness. Only for steeply sloped tapers, where the taper angle is about 30° or more, should heat flow trajectories based on the slope of the taper also be considered.

For small taper angles and one-way or four-way slopes, the true average R-value of the roof assembly may be found by multiplying the R-value for a flat insulation panel—whose thickness equals the average thickness of the tapered panel—by the percentage increase or decrease shown in Table 2. These modification factors or *efficiencies* depend only on the ratio of the high- and low-point thickness of the tapered panel. The "average" thickness may be taken as $(H + L)/2$ for both one- and four-way tapers; or the "equal-volume" measure of average height may be used, as long as the corresponding values of efficiency from Table 2 are selected. For triangular crickets, an intermediate thickness (R-value) ratio is also needed, as shown in Table 3.

Finally, large taper angles are not commonly encountered in practice. Where they occur, the efficiencies from Tables 2 and 3 should be multiplied by the reduction factors shown in Table 4, based on the taper slope. However, for commonly encountered taper slopes, it is not necessary to use Table 4.

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Table 4. Efficiency of Tapered Panels Comparing (Assumed) Vertical Heat Flow Trajectories versus (Actual) Curved Trajectories

Vertical distance in inches per ft of horizontal distance (slope expressed as ratio of vertical rise to horizontal run)	12 (1:1)	6 (1:2)	4 (1:3)	2 (1:6)	1 (1:12)	1/2 (1:24)	1/4 (1:48)
Angle (degrees)	45	26.6	18.4	9.5	4.8	2.4	1.2
Vertical vs. curved trajectory efficiency (%)	78.54	92.73	96.53	99.09	99.77	99.94	99.99

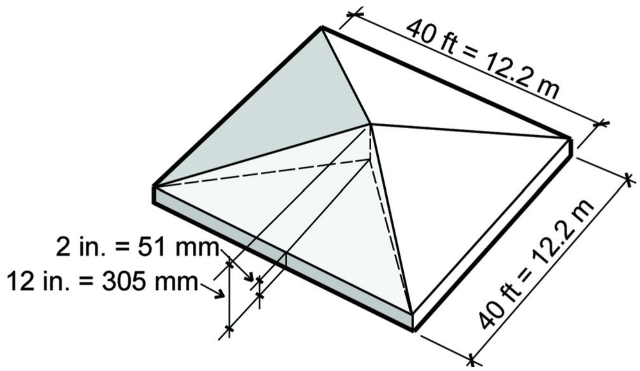

Figure 8 *Dimensions and geometry of tapered roof insulation with four-way slope and exterior drains for Example 1.*

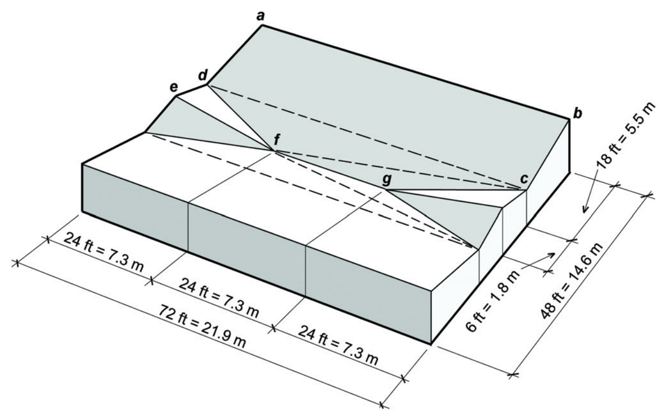

Figure 9 *Dimensions and geometry of tapered roof insulation with two-way slope and crickets for Example 2.*

Table 5. Computation of Heat Loss for Complex Roof Using Tapered Insulation with Crickets[1]

	Roof Components			
	One-Way Slope: a-b-c-d	**One-Way Slope down to a Point: c-d-f**	**One-Way Slope up to a Point: c-f-g**	**Cricket: d-e-f**
(a) High-point thickness	6.5 in. (165 mm)	2.5 in. (64 mm)	2.5 in. (64 mm)	4.5 in. (114 mm)
(b) Low-point thickness	2.5 in. (64 mm)	0.5 in. (13 mm)	0.5 in. (13 mm)	0.5 in. (13 mm)
(c) Intermediate-point thickness	—	—	—	2.5 in. (64 mm)
(d) R-value per in. of tapered insulation	6	6	6	6
(e) Additional constant R-value[2]	1	1	1	1
(f) Total R-value at high-point	6.5 × 6 + 1 = 40	2.5 × 6 + 1 = 16	2.5 × 6 + 1 = 16	4.5 × 6 + 1 = 28
(g) Total R-value at low-point	2.5 × 6 + 1 = 16	0.5 × 6 + 1 = 4	0.5 × 6 + 1 = 4	0.5 × 6 + 1 = 4
(h) Total R-value at intermediate-point	—	—	—	2.5 × 6 + 1 = 16
(i) Ratio of R-values at high- and low-point (row *f*/row *g*)	2.5	4.0	4.0	7.0
(j) Ratio of R-values at intermediate- and low-point (row *h*/row *g*)	—	—	—	4.0
(k) Efficiency, ε_a, of tapered insulation (Tables 2 and 3) based on rows *i* and *j*	94%	112%	71%	89%
(l) Assumed average R-value = (row *f* + row *g*)/2	(40 + 16)/2 = 28	(16 + 4)/2 = 10	(16 + 4)/2 = 10	(28 + 4)/2 = 16
(m) True average R-value = (row *l*) × (row *k*)	26.32	11.20	7.1	14.24
(n) True average U-factor = 1/(row *m*)	0.0380	0.0893	0.1408	0.0702
(o) Roof area of component × number of similar components (ft^2)	(72 × 18) × 2 = 2592	(1/2)(72 × 6) × 2 = 432	(1/2)(24 × 6) × 2 = 144	(1/2)(24 × 6) × 4 = 288
(p) Heat loss (Btu/h) = U-factor × roof area × DT = (row n) × (row o) × (70)	6895	2700	1419	1415
(q) Total heat loss (Btu/hr) from row *p*	6895 + 2700 + 1419 + 1415 = 12,429			
(r) Approximate heat loss (Btu/hr) based on average thickness of insulation applied over entire roof area[3]	Average insulation thickness = (0.5+6.5)/2 = 3.5 in. Total "average" R-value = 3.5 × 6 + 1 = 22 U-factor = 1/22 = 0.0455 Btu/h = (72 × 48)(0.0455) × 70 = 11,007			

1. Web-based calculators (Ochshorn 2010) can be used to compute heat loss for such complex roofs.
2. Thermal resistance of air films can be included in the calculation of additional constant R-value.
3. Approximate heat loss (row r) is based on "average" thickness equal to arithmetic average of high- and low-point insulation thickness, and does not account for actual tapered insulation efficiency.