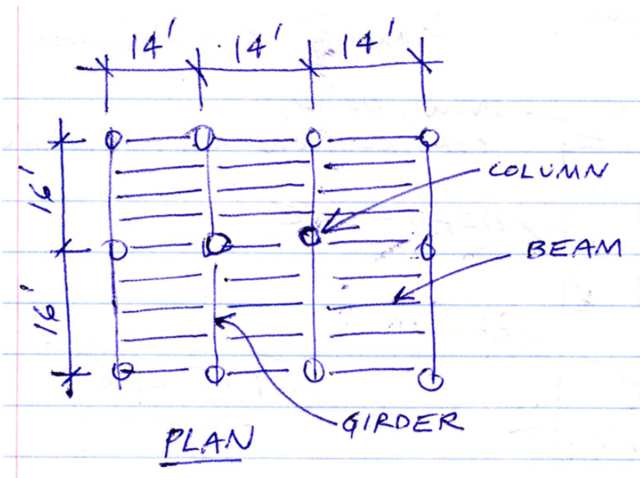
ARCH 2615-5615 Spring 2022: Assignment #2 Solutions to Part B

Problem definition. Design a Douglas Fir-Larch No. 2 girder using 4× lumber to support a residential live load as shown in Figure 3.24. Assume 10.5 psf for dead load. Loads on the girder can be modeled as being uniformly distributed since joists are spaced closely together.



Solution overview. Find loads; find known adjustments to allowable bending stress; use [Appendix Table A-3.16] to directly compute lightest cross section for bending; check for shear and deflection. Alternatively, begin iterative design process by assuming unknown adjustments to allowable stresses; then check bending stress (required section modulus), shear stress (required cross-sectional area) and deflection, as in analysis examples. Recompute if necessary with bigger (or smaller) cross section until bending, shear and deflection are OK.

Problem solution

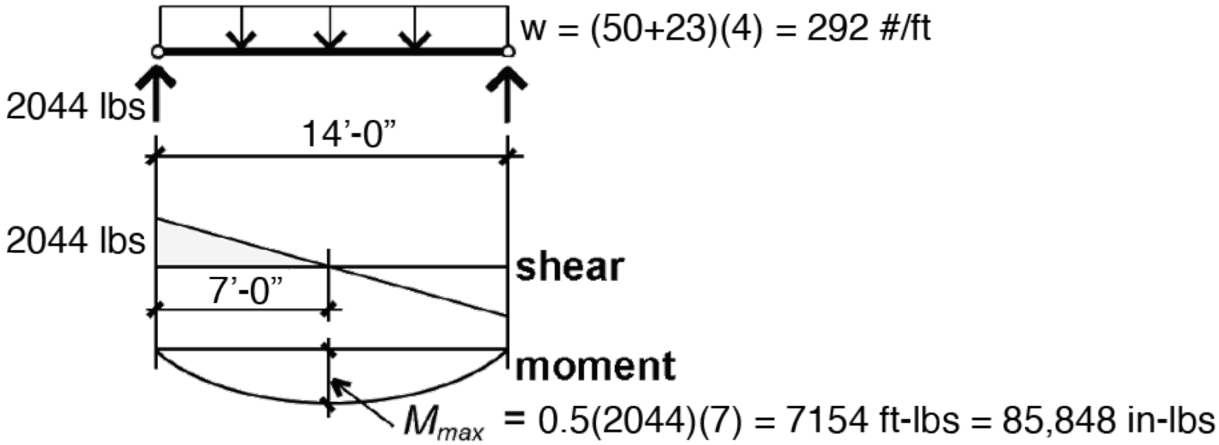
1. Find loads:

a. From [Appendix Table A-2.2], and given in assignment, the live load for an office occupancy, L = 50 psf.

b. The dead load, D = 23 psf (given).

c. The total distributed load, $w = (L + D)$(tributary area) = (50+23)(4) = 292 lb/ft. Live load reduction does not apply since K_{LL} times the tributary area is less than 400 ft^2. The tributary area for w is measured along one linear foot of the girder, in the direction of its span, as shown in the framing plan.

2. Create load, shear and moment diagrams to determine critical (i.e., maximum) shear force and bending moment.



3. Find partially-adjusted allowable bending stress:

a. From Appendix Table A-3.5, the design (tabular) value for bending stress, F_b = 900 psi.

b. From Appendix Table A-3.6, the following adjustments can be determined:

C_r = 1.0; C_M = 1.0; C_D= 1.0; C_L = 1.0 (assume continuous bracing by floor deck). The size factor, C_F, need not, and cannot, be determined at this point.

c. The adjusted value for bending stress, with all adjustments known except for C_F, is F_b'' = 900C_F psi (the double "prime" distinguishes this value from the fully adjusted value, F_b').

4. From Equation 1.24, compute the required section modulus: $S_{req} = M/F_b' = M/(900C_F)$ = 85,848/(900C_F). This can be rewritten as $C_F\,S_x = M/(900)$ = 85,848/(900) = 95.39 in^3.

5. Rather than doing several "trial" designs, it is possible to find the correct cross section for bending directly, by using a table of combined size factors (C_F) and section moduli (S_x) with the lightest values highlighted. In this method, the adjusted allowable stress is computed without the size factor, since C_F is combined with the section modulus in the table. Appendix Table A-3.16 indicates directly that the lightest 4× section for bending is a 4 × 14, based on a combined $C_F\,S_x$ value of 102.4 in^3, which is larger than the required value of $C_F\,S_x$ = 95.39 in^3 found in step 4.

6. Find adjusted allowable shear stress:

a. From Appendix Table A-3.7, the design (tabular) allowable shear stress F_v = 180 psi.

b. From Appendix Table A-3.8, there are no adjustments for shear stress; i.e.: C_M = 1.0; C_D = 1.0.

c. The adjusted value for allowable shear stress, F_v' = 180 psi.

7. Based on Equation 1.29, the required cross-sectional area to resist shear, A_{req} = $1.5V/F_v'$ = 1.5(2044)/180 = 17.03 in^2.

8. From Appendix Table A-3.12, we can check the actual area of the cross section, A_{act} = 46.38 in^2; since A_{act} = 46.38 in^2 ≥ A_{req} = 17.03 in^2, the section is OK for shear.

9. From Appendix Table A-1.3, find the allowable total-load deflection for a floor beam: $\Delta_{T_{allow}}$ = span/240 = (14 × 12)/240 = 0.7 in.; and the allowable live-load deflection for a floor joist: Δ_{allow} = span/360 = (14 × 12)/360 = 0.47 in.

10. From Appendix Table A-3.15, we can check the actual total-load deflection: $\Delta_{T_{act}} = CP(L/12)^3/(EI)$ where:

C = 22.46.

L = 14 × 12 = 168 in.

P = w(L/12) = (50 + 23)(4)(168/12) = 4088 lb.

$E = E'$ = 1,600,000 psi (from Appendix Table A-3.9).

I = 678.5 in^4 (directly from Appendix Table A-3.12, or from the equation, $I = bd^3/12$).

$\Delta_{T_{act}}$ = 22.46(4088)(168/12)^3/(1,600,000 × 678.5) = 0.23 in. Since $\Delta_{T_{act}}$ = 0.23 in. ≤ $\Delta_{T_{allow}}$ = 0.7 in., the beam is OK for total-load deflection.

11. From Appendix Table A-3.15, we can check the actual live-load deflection: $\Delta_{L_{act}} = CP(L/12)^3/(EI)$ where:

C = 22.46.

L = 14 × 12 = 168 in.

$P = w$(L/12) = (50 × 4)(168/12) = 2800 lb (Use live load only!).

$E = E'$ = 1,600,000 psi (from Appendix Table A-3.9).

I = 678.5 in^4 (directly from Appendix Table A-3.12, or from the equation, $I = bd^3/12$).

$\Delta_{L_{act}}$ = 22.46(2800)(168/12)^3/(1,600,000 × 678.5) = 0.16 in. Since $\Delta_{L_{act}}$ = 0.16 in. ≤ $\Delta_{L_{allow}}$ = 0.47 in., the beam is OK for deflection.

12. Conclusion: The 4 × 14 section is OK for bending, shear and deflection. Therefore it is acceptable.